PEBBLE ACCRETION BY BODIES ON ECCENTRIC ORBITS AND THE MASS RATIOS OF EXOPLANETS

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Introduction: In the last decade, several lines of evidence—from our own solar system and from studies of exoplanets—have converged to suggest that growth of planets is extraordinarily fast. In our own solar system, Hf-W dating of Mars shows it was an embryo formed between 1 and 3 Myr after CAIs (calcium-rich, aluminum-rich inclusions) [1]. The isotopic dichotomy of the solar system [2], reduced water content inside the snow line [3] and the concentration of CAIs in carbonaceous chondrites [4] have all been explained by invoking formation of Jupiter's ~20-30 M_E core in < 1 Myr. Ingassing of nebula gas into the magma oceans of the Earth [5,6] and Theia [7] explain their low-D/H hydrogen reservoirs and their He and Ne isotopic abundances. This requires them to achieve masses of several × 0.1 M_E in < 3 Myr. In exoplanetary systems, many super-Earths accreted substantial hydrogen-rich atmospheres [8,9], requiring them to grow to several Earth masses within < 3 Myr [10]. The gaps in young protoplanetary disks (sometimes ~ 10^5 yr, like HL Tau) observed by the Atacama Large Millimeter Array, may be due to massive (tens of Earth masses) planets [11]. Overall, planets in our solar system and others demand accretion rates of at least $10^{-6}\,M_E\,yr^{-1}$.

Recent astrophysical modeling suggests a path for rapidly turning micron-sized dust into planets thousands of km in diameter. Fast coagulation produces millimeter-sized particles (precursors of compactified dust, or melted chondrules). For conditions in the inner disk, their Stokes number (the ratio of aerodynamic stopping time to orbital time) is St ~ 10^{-3}. Turbulence then concentrates these into aggregates up to ~10 cm in size [12], with St ~ 0.01 – 0.1. These particles are intermittently but rapidly collected into ~100-km objects by streaming instability [13]. The very largest of these objects, those several ×100 km in diameter, can grow rapidly by pebble accretion. This aerodynamic process relies on drag to slow particles (especially those with St ~ 0.01 – 0.1) in the vicinity of a growing planet, allowing it to capture almost all such particles entering the planet's Hill sphere [14]. Growth at rates of at least $10^{-6}\,M_E\,yr^{-1}$ appear possible.

As successful as pebble accretion is, the theory appears incomplete at present. An important constraint from observations is that exoplanets in multiple-planet systems (observed in transits by Kepler and followed up by radial velocity measurements) appear to be similar in orbital period ratios and in size, and therefore also mass [15]. The growth of planets by pebble accretion in the Hill regime scales as $dM_p/dt \sim M_p^{2/3}$ [14], so the mass ratios between embryos should decrease with time very slowly, making it difficult to reconcile with the peas-in-a-pod result. We propose a modification of pebble accretion involving embryos on eccentric orbits that leads to embryos growing to similar sizes.

Growth on circular orbits: Growth of embryos on circular orbits proceeds as follows. Pebbles with St ~0.01-0.1 entering an embryo's Hill sphere with radius $R_H = a\,(M_p / 3M_\odot)^{1/3}$ are accreted with high efficiency, assuming they have scale height < R_H. Pebbles are assumed to sweep into the Hill sphere at velocity $V_H = R_H\Omega_K$, $\Omega_K = (GM_\odot/a^3)^{1/2}$, due to Keplerian shear across the Hill radius. The embryo sweeps up pebble mass at a rate $dM_p/dt = 2\,R_H\,\Sigma_p\,V_H$, where Σ_p is the surface density of pebbles (St~0.01–0.1 solids). This yields $dM_p/dt \sim 47\,(\Sigma_p / 1\,g\,cm^{-2})\,(M_p / 1\,M_E)^{2/3}\,M_E\,Myr^{-1}$ at 1 AU. This is such a high accretion rate, an embryo will quickly (~100 years) sweep up the mass in its torus, $2\pi a\,(2\,R_H)\,\Sigma_p = 0.0047\,(\Sigma_p / 1\,g\,cm^{-2})\,(M_p / 1\,M_E)^{1/3}\,M_E$, and will grow only if a pebble flux can feed its torus. [A surface density $\Sigma_p = 1\,g\,cm^{-2}$ of pebbles corresponds to 20% of all solids if the gas surface density is 1000 g cm^{-2}.] The radial drift of pebbles is $dM_p/dt = (2\pi a)\,\Sigma_p\,V_{pr}$, where $V_{pr} = -St\,(\eta V_K) / (1+St^2)$, and $\eta = -(C^2/ V_K^2)\,d\ln P / d\ln r$ [16]. But only a fraction of these, $(2R_H/V_{pr})(V_H/2\pi a)$ (which is < 1 if St ≥ 0.01 and $M_p \leq 1\,M_E$), are accreted before drifting through the annulus entirely, yielding the same result: $dM_p/dt = 2\,R_H\,\Sigma_p\,V_H$. Thus $dM_p/dt \sim M_p^{2/3}$ and larger planets should grow faster than small ones. Integrating the growth equation, assuming $\Sigma_p = 1\,g\,cm^{-2}$ at 1 AU, a 0.5 M_E planet would grow to 13.2 M_E in only 10^5 yr, and a 2.0 M_E planet would grow to 22.6 M_E in the same time.

Growth on eccentric orbits: Growth by pebble accretion is faster for an embryo on an eccentric orbit instead of a circular one. In a frame co-moving with the embryo, the embryo makes epicyclic orbits with radial excursions ±*ae*, where *e* is the eccentricity. The embryo and pebbles have relative velocity ~ $eV_K = ea\Omega_K$, several km/s. Analogous to the circular case, immediately after being put onto an eccentric orbit, embryos grow as $dM_p/dt = 2R_H\,\Sigma_p\,eV_K$, accreting a high fraction of the pebbles in its epicyclic torus in one orbit [17]. Thereafter it grows at the rate pebbles in the annulus between $a(1\text{-}e)$ and $a(1\text{+}e)$ can drift into the torus. Pressure support of gas makes it orbit at an azimuthal velocity $\eta V_K/2$ slower than the embryo, and the azimuthal velocities of pebbles differ from Keplerian by ~ $(-\eta V_K/2)$ / $(1 + St^2)$ [16]. Typically $\eta \sim 10^{-3}$; in the models of [4], $\eta = 3 \times 10^{-3}$. Within a time $2\pi a$ / $(\eta V_K/2)$ ~ $2\eta^{-1}$ orbits, the embryo sweeps up the entire annulus of area ≈ $2\pi a$ $(2ae)$. The mass accretion rate while it is sweeping up the annulus is $dM_p/dt = (ae)\ \Sigma_p\ (\eta V_K) / (1+St^2)$. A correction would have to be made if the Hill sphere were small enough that the pebbles could cross the torus width in less than one orbit, i.e., if $2R_H / (\eta V_K/2) < 2\pi / \Omega_K$, or $M_p < [157\ \eta]^3\ M_E \sim 0.0034\ M_E$; but as long as an embryo is at least this large, it will sweep up pebbles at the rate $dM_p/dt \approx (ae)\ \Sigma_p\ (\eta V_K)$. An embryo will sweep up the entire pebble mass of the annulus, $\Delta M = (2\pi a)(2ae)\ \Sigma_p = 0.05\ (\Sigma_p / 1\ \mathrm{g\ cm^{-2}})\ (e / 0.1)\ M_E$ in less than 10^3 yr.

As with circular orbits, continued embryo growth relies on radial drift of pebbles. Pebbles are brought into the annulus at a rate $dM_p/dt = (2\pi a)\ \Sigma_p\ V_{pr}$, where $V_{pr} = -St\ (\eta V_K)/(1+St^2)$. The fraction of these that are accreted is 100% if $St < e/2\pi$, and $dM_p/dt \approx 2\pi a\ \Sigma_p\ St\ \eta V_K$; but is $e/(2\pi St)$ if $St > e/2\pi$, and $dM_p/dt = ae\ \Sigma_p\ \eta V_K$, as before. Either generally exceeds the circular orbit pebble accretion rate = $2\ R_H\ \Sigma_p\ V_H$, because the embryo can sweep up pebbles from a larger area, and because pebbles are swept up with greater efficiency.

Embryos may accrete at the circular orbit rate until they are scattered onto an eccentric orbit. While on eccentric orbits, before their orbits are damped, an embryo can quickly accrete all of the pebbles in its annulus. At 1 AU in a disk model with gas densities like those of [4], $\tau = 0.1\ (M_p/0.5\ M_E)^{1/3}$ Myr for small embryos $M_p < 0.5\ M_E$ in the gas drag regime, and $\tau = 0.1\ (M_p/\ 0.5\ M_E)^{-1}$ Myr for larger embryos in the disk torque regime [18]. Embryos generally will accrete all the pebbles in their annulus before their eccentricities damp, gaining mass $\Delta M = (2\pi a)(2ae)\ \Sigma_p = 0.05\ (\Sigma_p / 1\ \mathrm{g\ cm^{-2}})\ (e / 0.1)\ M_E$ in $< 10^3$ yr. Thereafter they grow at rates dependent on the pebble flux. Integrating the coupled differential equations for growth dM_p/dt and damping $de/dt \sim -e\ M_p$, for $St > e/2\pi$, embryos grow linearly in time, but for durations that depend on their initial masses. A 0.5 M_E embryo set on an e=0.1 orbit will circularize in ~0.022 Myr and will grow to mass 10.1 M_E. A 2.0 M_E embryo will circularize in ~0.019 Myr and will grow to 10.3 M_E. In the case with $St < e/2\pi$, embryos may gain mass as their eccentricities damp, at rate $dM_p/dt \approx (ae)\ \Sigma_p\ (\eta V_K)$. Solving the coupled differential equations for growth $dM_p/dt \sim e$ and damping $de/dt \sim -e\ M_p$, we find that being scattered onto an orbit with eccentricity e, will cause a 0.5 M_E embryo to reach mass $M_p = 3.80\ (\Sigma_p / 1\ \mathrm{g\ cm^{-2}})^{1/2}\ (e / 0.1)^{1/2}\ M_E$, and a 2.0 M_E embryo to reach mass $M_p = 4.26\ (\Sigma_p / 1\ \mathrm{g\ cm^{-2}})^{1/2}\ (e / 0.1)^{1/2}\ M_E$, both within ~$5 \times 10^4$ yr. Once an embryo's orbit has circularized, it will continue to accrete at the circular pebble accretion rate, $dM_p/dt = 2\ R_H\ \Sigma_p\ V_H$.

System Architecture: In systems with embryos growing by pebble accretion on circular orbits, $dM_p/dt \sim M_p^{2/3}$, and the mass ratio of two embryos tends to decease with time, but slowly. In the example above, over 0.1 Myr, the mass ratio of two embryos decreased from (2.0)/(0.5) = 4.0, to (22.6)/(13.2) = 1.7. Embryos scattered onto eccentric orbits will accrete by pebble accretion more rapidly than if they were on circular orbits, at rates independent of mass, at least until their eccentricities damp. In the one example above, with $St > e/2\pi$, the mass ratio of two embryos decreased from (2.0)/(0.5) = 4.0, to (10.3)/(10.1) = 1.02. In the other case, with $St < e/2\pi$, the mass ratio decreased from (2.0)/(0.5) = 4.0, to (4.26)/(3.80) = 1.12. Embryos are only transiently on eccentric orbits, after being scattered; but while they are, the small embryos tend to grow more rapidly and catch up in mass with the larger embryos. By the time they each circularize, they will have attained masses much more similar than if they grew at the circular orbit pebble accretion rate. The tendency of exoplanets in the same system to have similar masses [8] may best be explained if a significant fraction of their growth occurs after they are scattered onto eccentric orbits.

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